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Torsion, Superconductivity, and Massive Electrodynamics

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A model for superconductor, based on massive electrodynamics in spaces with torsion is given. The generalized London equations for Cartan spaces are presented. From the London equations we show that it is possible to obtain an expression for the magnetic permeability constant in vacuum in terms of the time component of the torsion vector.

1. INTRODUCTION

The problem of massive electrodynamics in Cartan spaces with torsion has been recently considered (Garcia de Andrade, 1989, 1990*a*,*b*, and to appear) with special emphasis on photon mass creation via a torsion mechanism, much in the same way that massive particles can be created on a strongly curved background in general relativity. One of the main features of this theory is that torsion propagates in a way that can induce dielectric effects (Garcia de Andrade and Cinelli L. de Oliveira, 1990) in vacuum, analogously to how linearized Einstein gravity can induce electromagnetic effects in vacuum (de Sabbata and Gasperini, 1979). In this paper we apply this massive electrodynamics to build a model for superconductivity based on the construction of generalized London equations (London and London, 1935) in spaces with torsion. In the next section we present the wavelike equations for the electric and magnetic vectors **E** and **B**, respectively, and also derive the contributions of torsion to the dispersion relation, examining

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what happens in the case of the Meissner effect. This is not the first attempt to construct a model for superconductivity via torsion; Gregorash and Papini (1980) developed a model constructing a Weyl–Dirac theory with torsion.

2. SUPERCONDUCTIVITY IN CARTAN SPACES

Here we consider the electromagnetic field tensor as $F_{ij}=\partial_i A_j-\partial_j A_i$, where, although the photon does not couple with torsion under minimal coupling (between gravity and electromagnetism), it does couple in the non-minimally coupled constructed Lagrangian (Garcia de Andrade, 1989, 1990*a*, and to appear)

$$\mathscr{L}_{\mathrm{ECP}}=(-g)^{1/2}[R(\Gamma)(1+\lambda A^2)-\tfrac{1}{4}F_{ij}F^{ij}+J^i A_i] \tag{1}$$

Variation of this Lagrangian with respect to the 4-vector potential A^i ($i=0, 1, 2, 3$) yields the pair of Maxwell–Cartan–Proca equations

$$\partial_j F^{ij}=4\pi J'^i \tag{2}$$

$$\partial_{[i}F_{jk]}=0 \tag{3}$$

where the new 4-current $J'^i\equiv J^i-(3\lambda/4\pi k)(\partial_j Q^j)A^i$ contains an explicit contribution of torsion via the torsion 4-vector Q^j, where $Q^k_{ij}\equiv\Gamma^k_{[ij]}$ is the torsion tensor and Γ^i_{jk} is the U_4 Riemann–Cartan connection of spacetime. Equations (2) and (3) are formally identical to the Maxwell equations, although now they describe a massive electrodynamics through the term $\partial_i A^i\sim m^j_\gamma$, where m_γ represents the mass of the photon. Recent papers considering static massive photons (Chen-Yan, 1989) and the case of the transition between a massless to a massive photon (Stephens, 1989) near a collapsing massive neutron star give new insights into the problem following the early classical papers on the subject by Zeldovich (Wolf, 1990) and by Nieto and Goldhaber in the late 1970s. For simplicity and comparison with London's work, let us rewrite equations (2) and (3) in (**E**, **B**) form:

$$\nabla\cdot\mathbf{E}=4\pi\rho-\frac{3\lambda}{k}(\partial_0 Q^0+\nabla\cdot\mathbf{Q})\phi \tag{4}$$

$$\nabla\cdot\mathbf{B}=0 \tag{5}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t} \tag{6}$$

$$\nabla\times\mathbf{E}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{3\lambda}{k}(\partial_0 Q^0+\nabla\cdot\mathbf{Q})\mathbf{A}+\frac{4\pi}{c}\mathbf{J} \tag{7}$$

where for simplicity we consider a cosmological photon sea where $\mathbf{Q}=0$ and suppose that $Q^0 \equiv Q^0(t)$, which is a hypothesis connected with the rotation of the galaxy. Taking the curl of both sides of equation (7) yields

$$\nabla\times(\nabla\times\mathbf{B})=\frac{1}{c}\frac{\partial}{\partial t}(D\times\mathbf{E})-\frac{3\lambda}{k}(\partial_0 Q^0)\nabla\times\mathbf{A}+\frac{4\pi}{c}\nabla\times\mathbf{J} \qquad (8)$$

Using $\mathbf{B}=\nabla\times\mathbf{A}$ in (8) yields

$$\Box\mathbf{B}=\nabla^2\mathbf{B}-\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}=\frac{3\lambda}{k}\left(\frac{\partial Q^0}{\partial t}\right)\mathbf{B}+\frac{4\pi}{c}\nabla\times\mathbf{J} \qquad (9)$$

The condition

$$-\nabla\times\mathbf{J}=\frac{3\lambda}{4\pi k}\left(\frac{\partial Q^0}{\partial t}\right)\mathbf{B} \qquad (10)$$

leads to $\Box\mathbf{B}=0$.

Comparison of this equation with one of the London's equations, namely

$$-\nabla\times\mathbf{J}=\mu_0^{-1}\beta^{-2}\mathbf{B} \qquad (11)$$

where β is a constant and μ_0 is the magnetic permeability in vacuum, we have

$$\mu_0^{-1}=\frac{3\lambda}{4\pi k}\left(\frac{\partial Q^0}{\partial t}\right) \qquad (12)$$

which gives us an expression for the magnetic permeability μ_0 in terms of torsion. Now an analogous computation of the electric vector $\mathbf{E}$ lead us to the following equation for the electric oscillation:[4]

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=\frac{3\lambda}{k}\left(\frac{\partial Q^0}{\partial t}\right)\mathbf{E}-\frac{3\lambda}{k}\left(\frac{\partial^2 Q^0}{\partial t^2}\right)\mathbf{A} \qquad (13)$$

In this equation we can observe that two different types of current appear: the current due to the electrical conductivity $\mathbf{J}=\sigma\mathbf{E}$, where the conductivity is also given in terms of torsion as $\sigma\equiv(3\lambda/k)(\partial Q^0/\partial t)$, and a

[4]This equation is similar to Wolf's equation for the propagation of coupled scalar electromagnetic waves.

superconductivity current $\mathbf{J}_L$ due to the Meissner effect $\mathbf{J}_L = \lambda\mathbf{A}$, where

$$\lambda = \frac{3\lambda}{k}\left(\frac{\partial^2 Q^0}{\partial t^2}\right)$$

yields the dispersion relation

$$k^2 = \frac{\omega^2}{c^2} + \frac{3\lambda c^3}{8\pi G}\frac{\partial Q^0}{\partial t} \tag{14}$$

showing explicitly the torsion contribution to the dispersion relation.

A relation similar to equation (14) was obtained by de Sabbata and Gasperini (Wolf, 1988) on the basis of a massless Maxwell electrodynamics.

Also, the new 4-current J'^i defined above can be split as $(\rho', \mathbf{J}')$,

$$J'^0 \equiv \rho' = J^0 - \frac{3\lambda}{4\pi k}(\partial_k Q^k)\phi \tag{15}$$

$$\mathbf{J}' = \mathbf{J} - \frac{3\lambda}{4\pi k}(\partial_k Q^k)\mathbf{A} \tag{16}$$

where $J^0 \equiv \rho$ is the old charge density. It is also important to point out that in the Minkowski–Cartan case it is possible to show that the charge is conserved. Note that even in the vacuum case ($\rho = 0, \mathbf{J} = 0$) it is possible to construct a current induced by torsion,

$$\rho' = -\frac{3\lambda}{4\pi k}(\partial_k Q^k)\phi = -\frac{3\lambda}{4\pi k}(\partial_0 Q^0 + \nabla \cdot \mathbf{Q})\phi \tag{17}$$

$$J' = -\frac{3\lambda}{4\pi k}(\partial_k Q^k)\mathbf{A} = -\frac{3\lambda}{4\pi k}(\partial_0 Q^0 + \nabla \cdot \mathbf{Q})\mathbf{A} \tag{18}$$

For a constant photon mass m_γ one has

$$\frac{\partial^2 Q^0}{\partial t^2} \sim \frac{\partial m_\gamma}{\partial t} = 0 \tag{19}$$

and the Meissner term in equation (11) disappears. We also obtain the conservation equation

$$\nabla \cdot \mathbf{J}' + \frac{\partial \rho'}{\partial t} = 0 \tag{20}$$

However, in the case of a nonconstant photon mass the rate of creation of mass is given by

$$\frac{\partial m_\gamma}{\partial t}=\frac{3\lambda}{4\pi k}\left(\frac{\partial^2 Q^0}{\partial t^2}\right)\phi \tag{21}$$

which shows explicitly that the torsion vector interacting with the electric potential ϕ is responsible for this massive photon production.

In the gravitational exact case it is possible that the cosmological constant may play an important role in the massive photon production. Let us now recall that the Proca-type equation ($\hbar=c=1$ in the relativistic units)

$$(\Box+m_\gamma^2)A^i=0 \tag{22}$$

where m_γ is not a constant, is not necessarily a Proca field equation. Let us also recall that in our case $Q^0=Q^0(t)$ and $\mathbf{Q}=0$, which reduces the above equation for charge and current to

$$\rho'=-\frac{3\lambda}{4\pi k}(\partial_0 Q^0)\phi \tag{23}$$

$$J'=-\frac{3\lambda}{4\pi k}(\partial_0 Q^0)\mathbf{A} \tag{24}$$

This last current is in fact the Meissner superconductivity current. Let us now investigate what sort of constraint must be imposed on the torsion vector to have conservation of current in the above sense.

To this end, let us compute $\nabla\cdot\mathbf{J}'$ and $\partial\rho'/\partial t$ separately. From equations (22) and (23) we have

$$\frac{\partial\rho'}{\partial t}=-\frac{3\lambda}{4\pi k}\left\{\left(\frac{\partial^2 Q^0}{\partial t^2}\right)\phi+\left(\frac{\partial Q^0}{\partial t}\right)\left(\frac{\partial\phi}{\partial t}\right)\right\} \tag{25}$$

$$\nabla\cdot\mathbf{J}'=-\frac{3\lambda}{4\pi k}\left\{\left(\frac{\partial Q^0}{\partial t}\right)\nabla\cdot\mathbf{A}\right\} \tag{26}$$

Addition of equations (24) and (25) yields

$$\frac{\partial\rho'}{\partial t}+\nabla\cdot\mathbf{J}'=-\frac{3\lambda}{4\pi k}\left\{\frac{\partial Q^0}{\partial t}\left[\nabla\cdot\mathbf{A}+\frac{\partial\phi}{\partial t}\right]+\left(\frac{\partial^2 Q^0}{\partial t^2}\right)\phi\right\} \tag{27}$$

However, the Lorentz condition $\nabla\cdot\mathbf{A}+\partial\phi/\partial t=0$ reduces equation (26) to

$$\frac{\partial\rho'}{\partial t}+\nabla\cdot\mathbf{J}'=-\frac{3\lambda}{4\pi k}\left(\frac{\partial^2 Q^0}{\partial t^2}\right)\phi \tag{28}$$

So if we recall our previous discussion on the constancy of the photon mass we see that equation (26) simply expresses the fact that the current is conserved in the case of constant photon mass, or, in other words, variation of the photon mass breaks out only gauge invariance but also charge conservation, which is a serious problem for physics.

3. CONCLUSIONS

We have shown that a torsion vector field can induce oscillations in the electric and magnetic fields **E** and **B** in a Minkowski spacetime background with torsion. This reasoning also leads to generalized London equations in Cartan space. We are considering the computation of the force between two beams of massive photons and studying the change in polarization due to the torsion field. There is also a promising application of the ideas discussed here in plasma physics and astrophysics.

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